IOWA STATE UNIVERSITY

ECpE Department

Introduction to Power Flows in Distribution Systems

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Overview

The power-flow analysis of a distribution feeder is different from that of an interconnected transmission system due to its radiality and unbalance. Typically, what will be known prior to the analysis will be the three-phase voltages at the substation and the complex power of all loads and the load models (constant complex power, constant impedance, constant current, or a combination).

- A power-flow analysis of a feeder can determine the following:
  - Voltage magnitudes and angles at all nodes of the feeder
  - Line flow in each line section specified in kW and kvar, amps and degrees, or amps and power factor
  - Power loss in each line section
  - Total feeder input kW and kvar
  - Total feeder power losses
  - Load kW and kvar based upon the specified model for the load

Overview

To sum up, to perform power flow analysis in the distribution systems:

Known Quantities:

- 3Ø Voltage at substation
- Complex power (P, Q) at each node
- Grid topology + Impedance at line sections

Unknow Quantities (Want to Calculate)

- Voltage magnitude & angle at every node

Models for Power Flow

Because a distribution feeder is radial, iterative techniques commonly used in transmission network power-flow studies are not used because of poor convergence characteristics [1]. Instead, an iterative technique specifically designed for a radial system is used.

Methods to carry out power flow in distribution systems can be classified into two categories:

- Bus Injection Model
- Branch – Flow Model (Forward – Backward Sweep Method)

Bus Injection Model

- Some electrical distribution systems simulation software, such as OpenDSS uses Bus Injection Model to perform power flow analysis.
- The net current inject is given by,

$$I = Y.V \text{ ; } where,$$

$'Y'$ is Admittance Matrix (Combination of self & mutual admittance for each bus).

- **Note:** Y is a symmetric matrix, i.e. $Y_{ij} = Y_{ji}$
- For a bus 'i' :

$$Y_{ii} = y_i + \sum_{k=1, k\neq i}^{N} y_{ik} \text{ ; }$$

(the self-admittance of bus $'i'$ plus the sum of the admittances connecting to bus $'i'$

and, $Y_{ij} = -y_{ji}$

Note: In transmission system, we use Gauss Seidel and Newton – Raphson iterative methods to perform power flow analysis.

Bus Injection Model

In distribution Systems,

$$[I] = [Y]\ [V]$$

Algorithm:

1. Initialization:

No – Load Condition:

$$I_{source} = Y_{source} \cdot V_{source} \quad [here, V_{source} \text{ is a substation voltage}]$$

2. $V^{(0)} = \left[Y_{system}\right]^{-1} \cdot I_{source}$

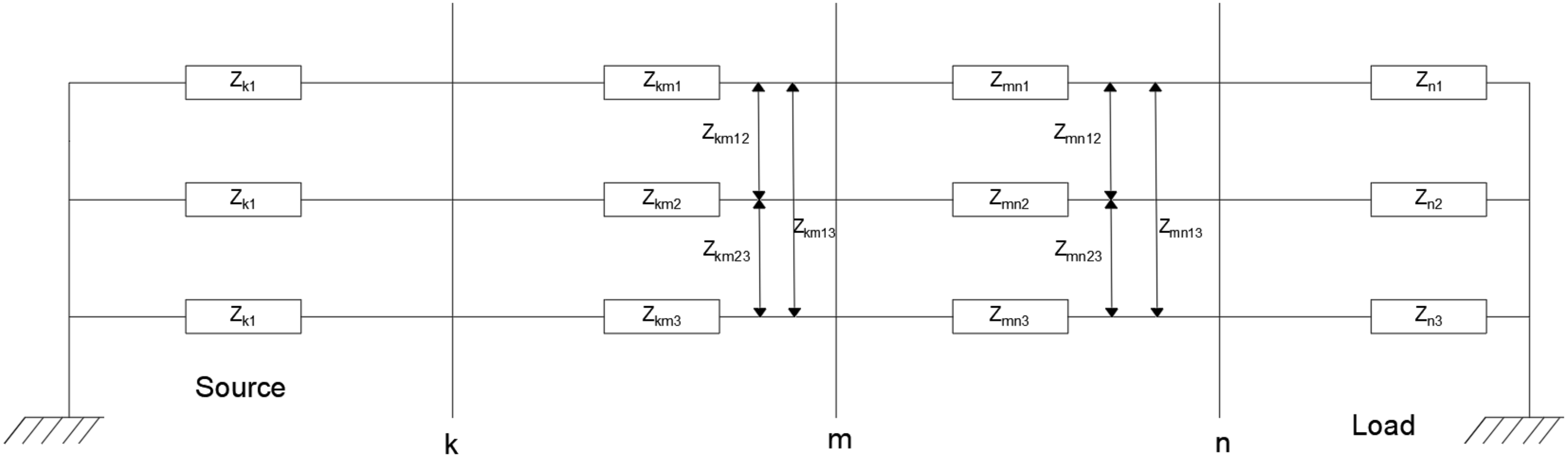
3. Update $[I]$

4. Update $[V]^k$

$$[V]^k = [Y]^{-1}[I]^k$$

5. Stop at $\left(V^{k+1} - V^k\right) < \varepsilon$

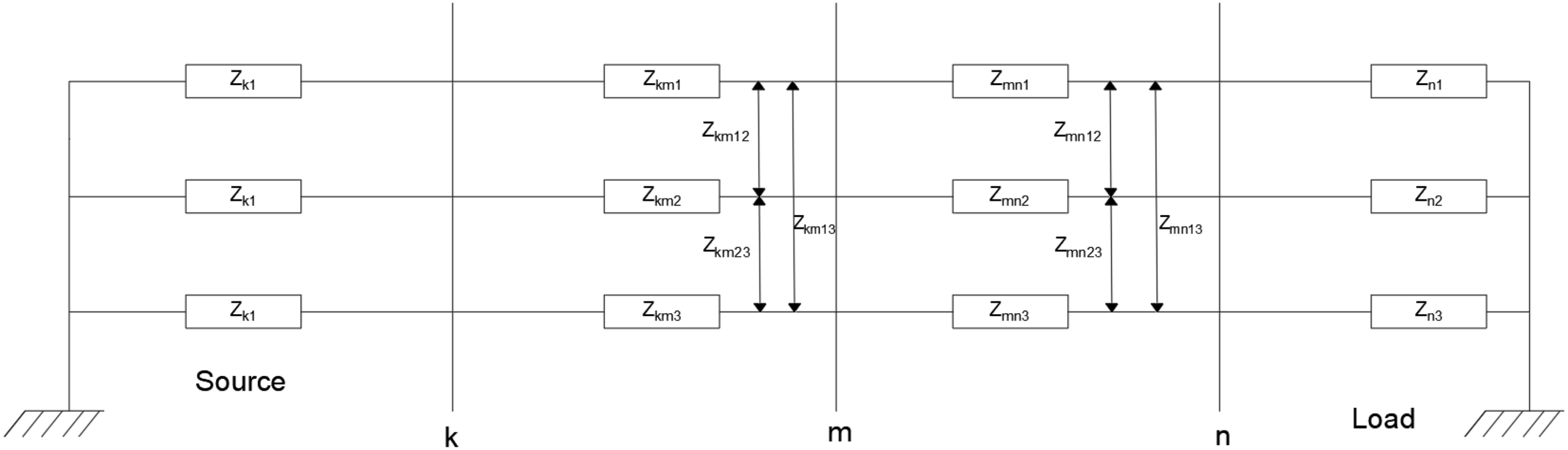
Bus Injection Model: How to build [Y]?



- $Z_k = \begin{bmatrix} z_{k_1} & 0 & 0 \\ 0 & z_{k_2} & 0 \\ 0 & 0 & z_{k3} \end{bmatrix}$ (The source impedance is provided)

- $Y_k = Z_k^{-1} = \begin{bmatrix} y_{k_1} & 0 & 0 \\ 0 & y_{k_2} & 0 \\ 0 & 0 & y_{k_3} \end{bmatrix}$

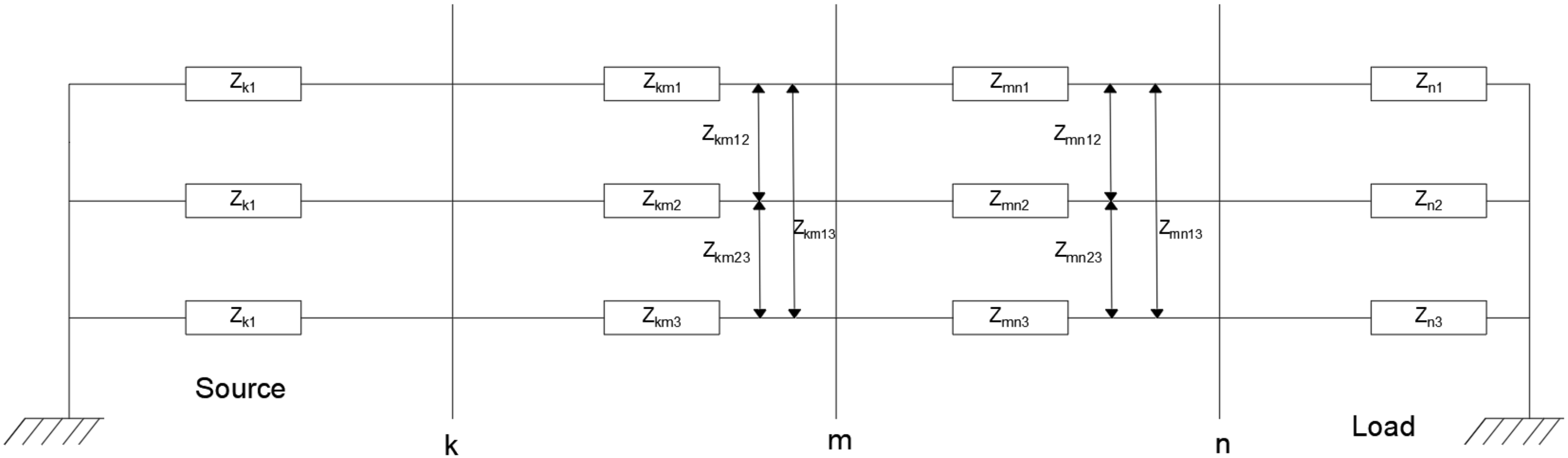
Bus Injection Model: How to build [Y]?



For k – m:

- $$Z_{km} = \begin{bmatrix} z_{km_1} & z_{km_{12}} & z_{km_{13}} \\ z_{km_{12}} & z_{km_2} & z_{km_{23}} \\ z_{km_{13}} & z_{km_{23}} & z_{km_3} \end{bmatrix}$$
- $$Y_{km} = Z_{km}^{-1} = \begin{bmatrix} y_{km_1} & y_{km_{12}} & y_{km_{13}} \\ y_{km_{12}} & y_{km_2} & y_{kn_{23}} \\ y_{km_{13}} & y_{km_{23}} & y_{km_3} \end{bmatrix}$$

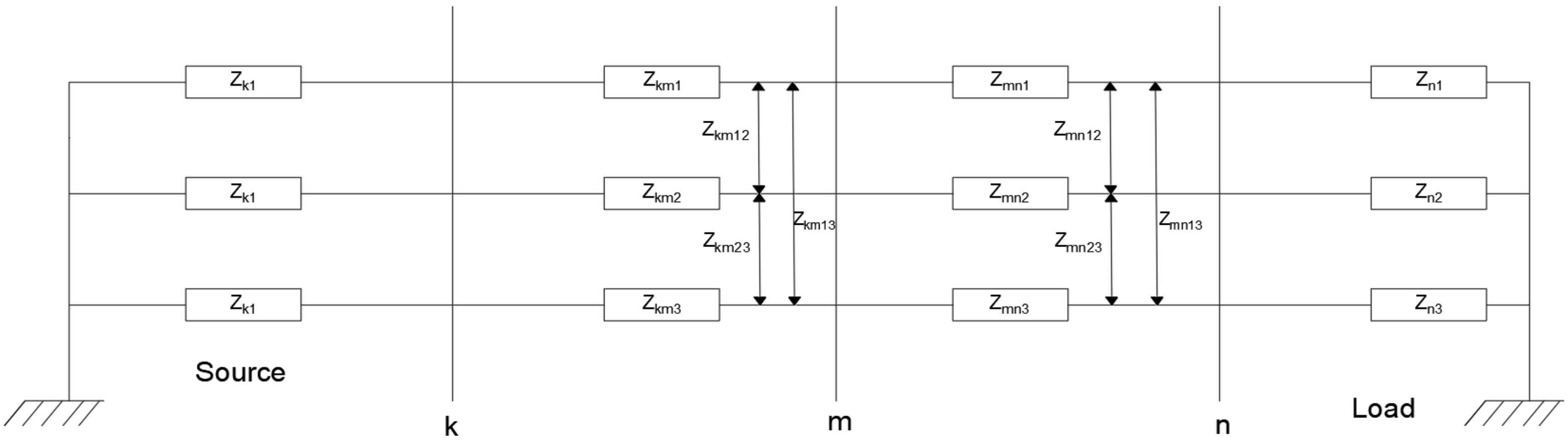
Bus Injection Model: How to build [Y]?



For m-n section:

- $$z_{mn} = \begin{bmatrix} z_{mn_1} & z_{mn_{12}} & z_{mn_{13}} \\ z_{mn_{12}} & z_{mn_2} & z_{mn_{23}} \\ Z_{mn_{13}} & z_{mn_{23}} & z_{mn_3} \end{bmatrix}$$
- $$Y_{mn} = Z_{mn}^{-1} = \begin{bmatrix} y_{mn_1} & y_{mn_{12}} & y_{mn_{13}} \\ y_{mn_{12}} & y_{mn_2} & y_{mn_{23}} \\ y_{mn_{13}} & y_{mn_{23}} & y_{mn_3} \end{bmatrix}$$

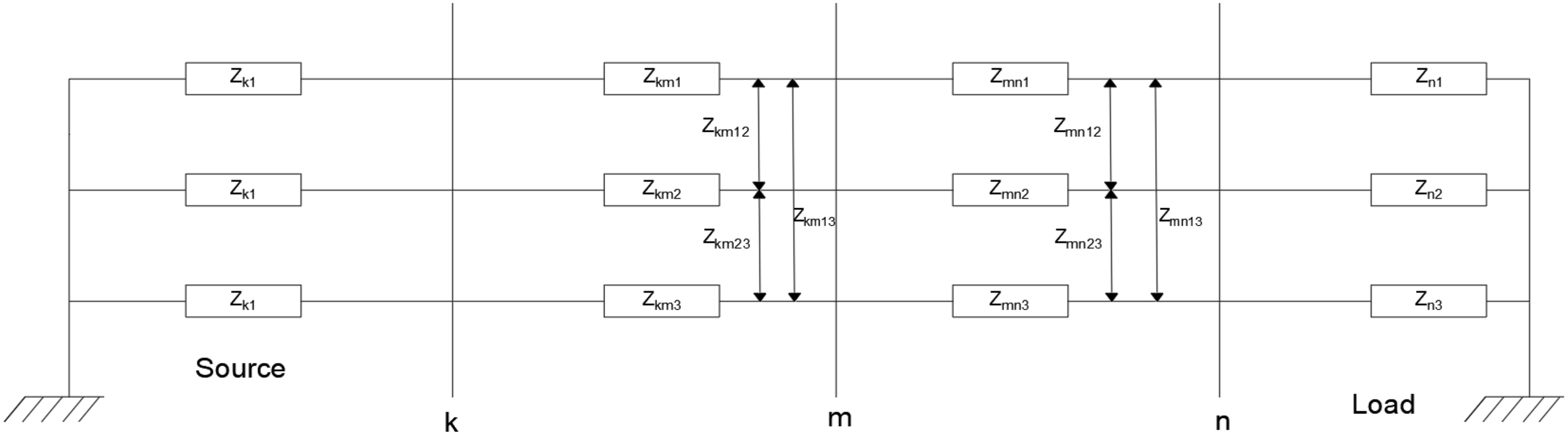
Bus Injection Model: How to build [Y]?



Load:

- $$Z_n = \begin{bmatrix} z_{n_1} & 0 & 0 \\ 0 & z_{n_2} & 0 \\ 0 & 0 & z_{n_3} \end{bmatrix}$$
- $$Y_n = Z_n^{-1} = \begin{bmatrix} y_{n_1} & 0 & 0 \\ 0 & y_{n_2} & 0 \\ 0 & 0 & y_{n_3} \end{bmatrix}$$

Bus Injection Model: How to build [Y]?



- The overall [Y] matrix is,

$$Y = \begin{bmatrix} Y_k + Y_{km} & -Y_{km} & 0 \\ -Y_{km} & Y_{km} + Y_{mn} & -Y_{mn} \\ 0 & -Y_{mn} & Y_{mn} + Y_n \end{bmatrix}$$

Branch Flow model

- Some electrical distribution systems simulation software, such as Milsoft Windmill uses Branch Flow Model to perform power flow analysis.

- A distribution feeder can be broken into the “series” components and the “shunt” components. These series components can be lines, transformers (three-phase substation transformers, single-phase/three-phase distribution transformers), voltage regulators...

- The shunt components of a distribution feeder are
  - Spot static loads
  - Spot induction machines
  - Capacitor banks

How to model Series Component?

- Figure below is a general model of series component; no distinction is made as to what type of element is connected between nodes.

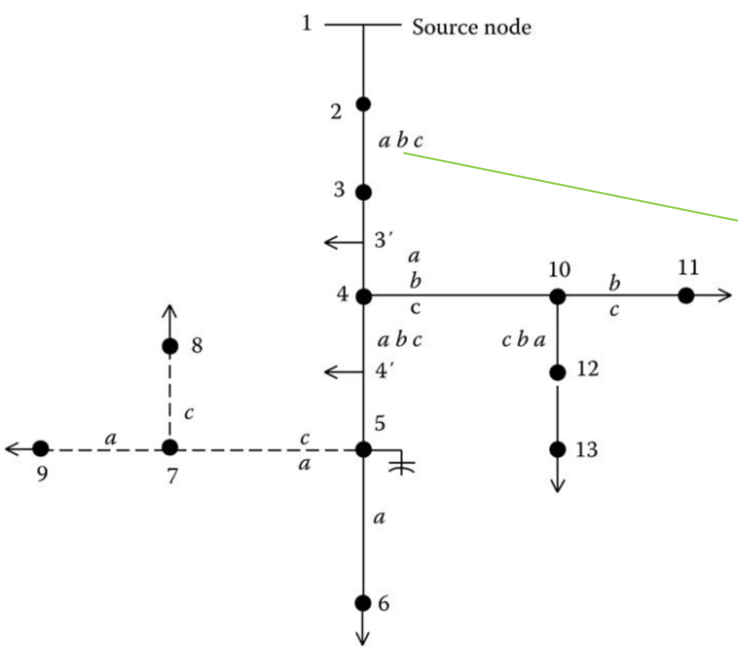


Fig.1 A typical unbalanced distribution feeder

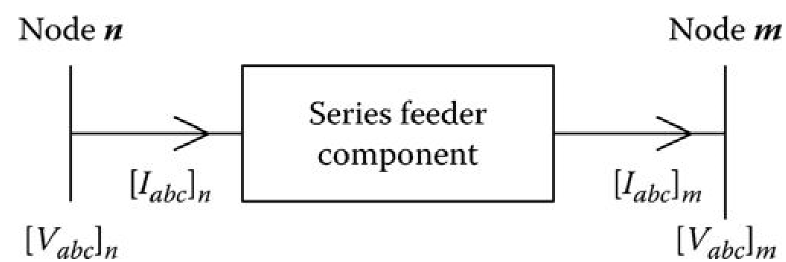
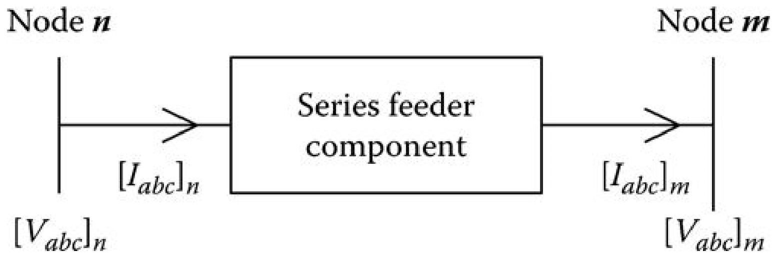


Fig.2 Standard feeder series component model

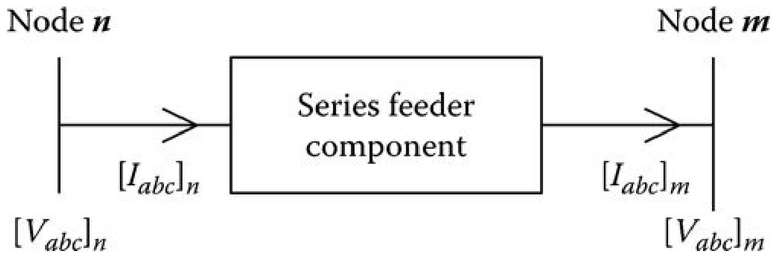
How to model Series Component?



$$[V_{abc}]_n = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_n \qquad [I_{abc}]_m = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_m$$

- How to relate $[V]_n$ with $[V]_m$?
- How to relate $[I]_n$ with $[I]_m$?

How to model Series Component?



- How to relate $[V]_n$ with $[V]_m$?
- How to relate $[I]_n$ with $[I]_m$?

- With reference to above figure, for any series components, they are modeled using the following two equations.
- These two equations are also known as forward and backward sweep models.

Forward sweep: $[VLN_{abc}]_m = [A] \cdot [VLN_{abc}]_n - [B] \cdot [I_{abc}]_n$ (1)

(Update Voltage)

Backward sweep: $[I_{abc}]_n = [c] \cdot [VLN_{abc}]_m + [d] \cdot [I_{abc}]_m$ (2)

(Update Current)

-Load Current

-KCL junction

Note: backward sweep is done from end node.

- For different components, the equations have the same format, and [A], [B], [c], and [d] are all 3×3 matrices.

How to connect each & every node section?

Forward sweep: $[VLN_{abc}]_m = [A] \cdot [VLN_{abc}]_n - [B] \cdot [I_{abc}]_n$ (1)

Backward sweep: $[I_{abc}]_n = [c] \cdot [VLN_{abc}]_m + [d] \cdot [I_{abc}]_m$ (2)

For line segment:

$$[A] = [u]$$

$$[B] = [z_{abc}] \longrightarrow \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \Omega\ /mile * length$$

$$[c] = [0]$$

$$[d] = [u]$$

How to connect each & every node section?

Forward sweep: $[VLN_{abc}]_m = [A] \cdot [VLN_{abc}]_n - [B] \cdot [I_{abc}]_n$ (1)

Backward sweep: $[I_{abc}]_n = [c] \cdot [VLN_{abc}]_m + [d] \cdot [I_{abc}]_m$ (2)

For Transformer (△ Grounded − Y Step-down):

$$[A] = \frac{1}{n_t}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[B] = \begin{bmatrix} Z_{t_a} & 0 & 0 \\ 0 & Z_{t_b} & 0 \\ 0 & 0 & Z_{t_c} \end{bmatrix}$$

Where, $n_t = \frac{VLL_{rated\ primary}}{VLN_{rated\ secondary}}$

$$[c] = [0]$$

$$[d] = \frac{1}{n_t}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

How to connect each & every node section?

Forward sweep: $[VLN_{abc}]_m = [A] \cdot [VLN_{abc}]_n - [B] \cdot [I_{abc}]_n$ (1)

Backward sweep: $[I_{abc}]_n = [c] \cdot [VLN_{abc}]_m + [d] \cdot [I_{abc}]_m$ (2)

For Volage Regulator ($In - Line\ VR\ or\ Step\ VR$):

$$[A] = \begin{bmatrix} 1/a_{R_a} & 0 & 0 \\ 0 & 1/a_{R_b} & 0 \\ 0 & 0 & 1/a_{R_c} \end{bmatrix}$$

$$[B] = [0]$$

$$[c] = [0]$$

$$[d] = \begin{bmatrix} 1/a_{R_a} & 0 & 0 \\ 0 & 1/a_{R_b} & 0 \\ 0 & 0 & 1/a_{R_c} \end{bmatrix}$$

- Voltage Regulator is also modelled as auto transformer.
- Voltage Regulator is 3-phase and has Y-Y connection.
- $a_R = 1 \pm 0.00625 \cdot \text{Tap}$

How to connect each & every node section?

Forward sweep: $[VLN_{abc}]_m = [A] \cdot [VLN_{abc}]_n - [B] \cdot [I_{abc}]_n$ (1)

Backward sweep: $[I_{abc}]_n = [c] \cdot [VLN_{abc}]_m + [d] \cdot [I_{abc}]_m$ (2)

For Volage Regulator ($In - Line\ VR\ or\ Step\ VR$):

$$[A] = \begin{bmatrix} 1/a_{R_a} & 0 & 0 \\ 0 & 1/a_{R_b} & 0 \\ 0 & 0 & 1/a_{R_c} \end{bmatrix}$$

$$[B] = [0]$$

$$[c] = [0]$$

$$[d] = \begin{bmatrix} 1/a_{R_a} & 0 & 0 \\ 0 & 1/a_{R_b} & 0 \\ 0 & 0 & 1/a_{R_c} \end{bmatrix}$$

- Voltage Regulator is also modelled as auto transformer.
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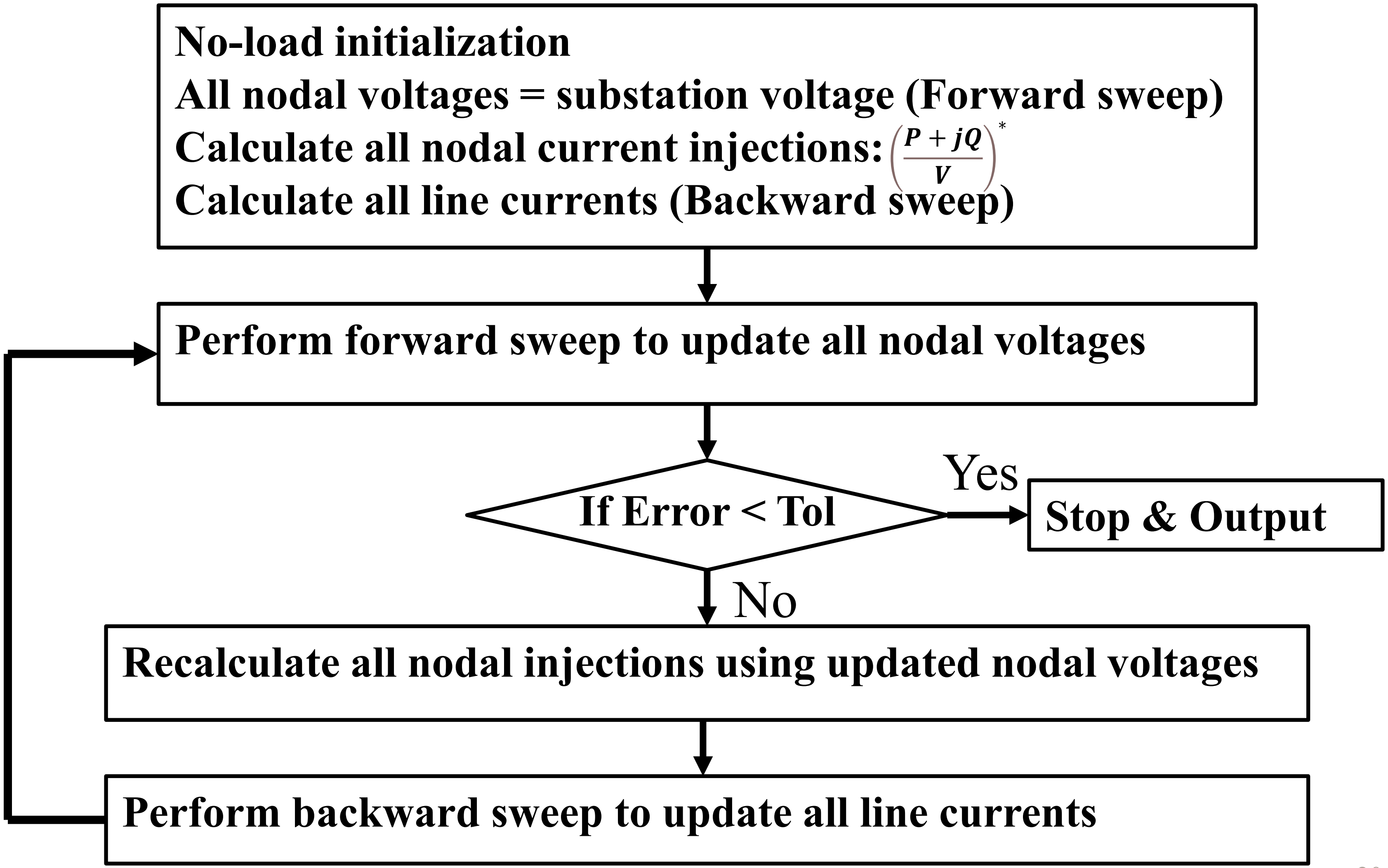
How to connect each & every node section?



Transformer model

	$\triangle$ − Grounded Y Step-down	Wye-Connected Voltage Regulator	Line Segment
$[at]$	$-\frac{n_t}{3}\begin{bmatrix}0 & 2 & 1\\1 & 0 & 2\\2 & 1 & 0\end{bmatrix}$	$\begin{bmatrix}a_{R\_a} & 0 & 0\\0 & a_{R\_b} & 0\\0 & 0 & a_{R\_c}\end{bmatrix}$	$[u]$
$[bt]$	$-\frac{n_t}{3}\begin{bmatrix}0 & 2Z_{t_b} & Z_{t_c}\\Z_{t_a} & 0 & 2Z_{t_c}\\2Z_{t_a} & Z_{t_b} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$[Z_{abc}]$
$[ct]$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$
$[dt]$	$\frac{1}{n_t}\begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\begin{bmatrix}1/a_{R\_a} & 0 & 0\\0 & 1/a_{R\_b} & 0\\0 & 0 & 1/a_{R\_c}\end{bmatrix}$	$[u]$
$[At]$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & -1\\-1 & 1 & 0\\0 & -1 & 1\end{bmatrix}$	$\begin{bmatrix}1/a_{R\_a} & 0 & 0\\0 & 1/a_{R\_b} & 0\\0 & 0 & 1/a_{R\_c}\end{bmatrix}$	$[u]$
$[Bt]$	$\begin{bmatrix}Z_{t_a} & 0 & 0\\0 & Z_{t_b} & 0\\0 & 0 & Z_{t_c}\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$[Z_{abc}]$
	$n_t = \frac{VLL_{rated\ primary}}{VLN_{rated\ secondary}}$	$a_R = 1 \pm 0.00625 \cdot \text{Tap}$	

$$[VLG_{abc}]_m = [A] \cdot [VLG_{abc}]_n - [B] \cdot [I_{abc}]_m$$

$$[I_{abc}]_n = [c] \cdot [VLG_{abc}]_m + [d] \cdot [I_{abc}]_m$$

$$[VLG_{abc}]_n = [a] \cdot [VLG_{abc}]_m + [b] \cdot [I_{abc}]_m$$

Transformer model

	$\triangle -$ Grounded Y Step-down	$\triangle -$ Grounded Y Step-up	Ungrounded Y $- \triangle$ Step-down	Grounded Y $-$ Grounded Y
$[a_t]$	$-\frac{n_t}{3}\begin{bmatrix}0&2&1\\1&0&2\\2&1&0\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix}$	$n_t\begin{bmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{bmatrix}$	$\begin{bmatrix}n_t&0&0\\0&n_t&0\\0&0&n_t\end{bmatrix}$
$[b_t]$	$-\frac{n_t}{3}\begin{bmatrix}0&2Z_{t_b}&Z_{t_c}\\Z_{t_a}&0&2Z_{t_c}\\2Z_{t_a}&Z_{t_b}&0\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}2Z_{t_a}&Z_{t_b}&0\\0&2Z_{t_b}&Z_{t_c}\\Z_{t_a}&0&2Z_{t_c}\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}Z_{t_{ab}}&-Z_{t_{ab}}&0\\Z_{t_{bc}}&2Z_{t_{bc}}&0\\-2Z_{t_{ca}}&Z_{t_{ca}}&0\end{bmatrix}$	$\begin{bmatrix}n_tZ_{t_a}&0&0\\0&n_tZ_{t_b}&0\\0&0&n_tZ_{t_c}\end{bmatrix}$
$[c_t]$	$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$
$[d_t]$	$\frac{1}{n_t}\begin{bmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1&0&-1\\0&1&-1\\-1&0&1\end{bmatrix}$	$\frac{1}{3n_t}\begin{bmatrix}1&-1&0\\1&2&0\\-2&-1&0\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
$[A_t]$	$\frac{1}{n_t}\begin{bmatrix}1&0&-1\\-1&1&0\\0&-1&1\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1&0&-1\\0&1&-1\\-1&0&1\end{bmatrix}$	$\frac{1}{3n_t}\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
$[B_t]$	$\begin{bmatrix}Z_{t_a}&0&0\\0&Z_{t_b}&0\\0&0&Z_{t_c}\end{bmatrix}$	$[Z_{t_{abc}}]$	$\frac{1}{9}\begin{bmatrix}2Z_{t_{ab}}+Z_{t_{bc}}&2Z_{t_{bc}}-2Z_{t_{ab}}&0\\2Z_{t_{bc}}-2Z_{t_{ca}}&4Z_{t_{bc}}-Z_{t_{ca}}&0\\Z_{t_{ab}}-4Z_{t_{ca}}&-Z_{t_{ab}}-2Z_{t_{ca}}&0\end{bmatrix}$	$[Z_{t_{abc}}]$
n_t	$\frac{VLL_{rated\ primary}}{VLN_{rated\ secondary}}$	$\frac{VLL_{rated\ primary}}{VLN_{rated\ secondary}}$	$\frac{VLN_{rated\ primary}}{VLL_{rated\ secondary}}$	$\frac{VLN_{rated\ primary}}{VLN_{rated\ secondary}}$

Transformer model

	open Y − open △	△−△	Open △ − Open △
$[at]$	$[AV][D]_{(1)}$	$[w][AV][D]_{(2)}$	$[w][AV][D]$
$[bt]$	$\begin{bmatrix} n_t Z_{t_{ab}} & 0 & 0 \\ 0 & 0 & -n_t Z_{t_{bc}} \\ 0 & 0 & 0 \end{bmatrix}$	$[AV][w][Z_{t_{abc}}][G1]_{(3)}$	$[w][BI]_{(5)}$
$[ct]$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$[dt]$	$\frac{1}{n_t}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	$[AI]^{-1}{}_{(4)}$	$\frac{1}{n_t}\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$
$[At]$	$\frac{1}{3n_t}\begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ -1 & -2 & 0 \end{bmatrix}$	$[w][AV]^{-1}[D]$	$[w][BV][D]_{(6)}$
$[Bt]$	$\frac{1}{3}\begin{bmatrix} 2Z_{t_{ab}} & 0 & -Z_{t_{bc}} \\ -Z_{t_{ab}} & 0 & -Z_{t_{bc}} \\ -Z_{t_{ab}} & 0 & 2Z_{t_c} \end{bmatrix}$	$[w][Z_{t_{abc}}][G1]$	$[w][BI]$
	$n_t = \frac{VLN_{rated\ primary}}{VLL_{rated\ secondary}}$	$n_t = \frac{VLL_{rated\ primary}}{VLL_{rated\ secondary}}$	$n_t = \frac{VLL_{rated\ primary}}{VLL_{rated\ secondary}}$

Transformer model

$$(1)\ [AV] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(3)\ [F] = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ Z_{t_{ab}} & Z_{t_{bc}} & Z_{t_{ca}} \end{bmatrix}$$

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[G] = [F]^{-1}$$

$$[G1] = \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix}$$

$$(2)\ [w] = \frac{1}{3} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$(5)\ [BI] = n_t \begin{bmatrix} Z_{t_{ab}} & 0 & 0 \\ 0 & 0 & Z_{t_{bc}} \\ -Z_{t_{ab}} & 0 & Z_{t_{bc}} \end{bmatrix}$$

$$(4)\ [AI] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(6)\ [BV] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Series Components

- How to get $[A], [B], [\mathrm{c}], [d]$ for different components ?

- How to get $[Z_{abc}]\Omega/$ mile ?

Generalized Matrices

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \quad (5)$$

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \quad (6)$$

$$[I_{ABC}] = [c_t] \cdot [VLN_{abc}] + [d_t] \cdot [I_{abc}] \quad (7)$$

In Equations (5) through (7), the matrices $[VLN_{ABC}]$ and $[VLN_{abc}]$ represent the line-to-neutral voltages for an ungrounded wye connection or the line-to-ground voltages for a grounded wye connection.

For a delta connection, the voltage matrices represent "equivalent" line-to-neutral voltages. The current matrices represent the line currents regardless of the transformer winding connection.

In the modified ladder technique, Equation (5) is used to compute new node voltages downstream from the source using the most recent line currents. In the backward sweep, only Equation (7) is used to compute the source-side line currents using the newly computed load-side line currents.

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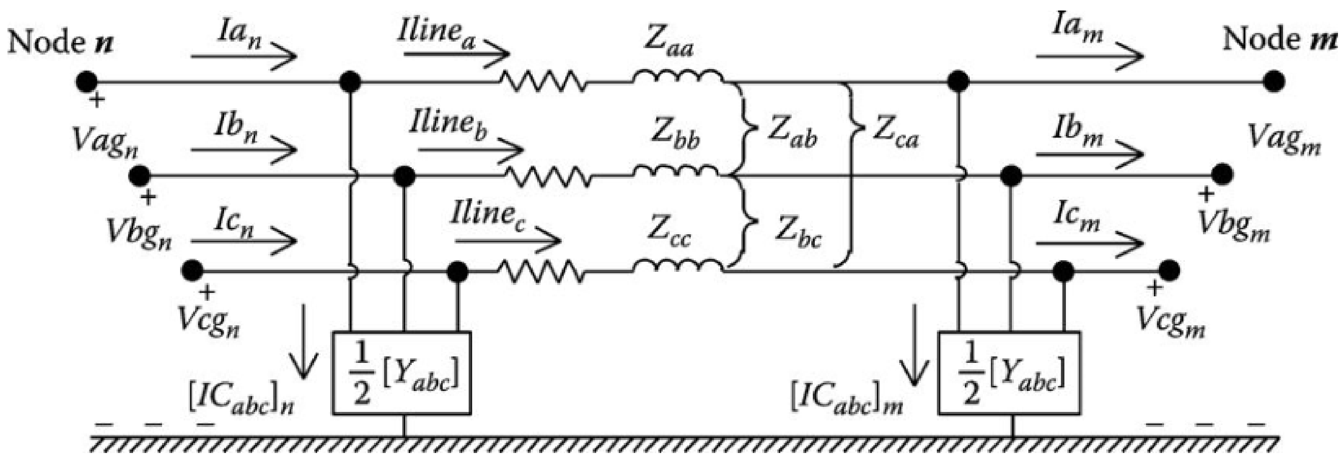
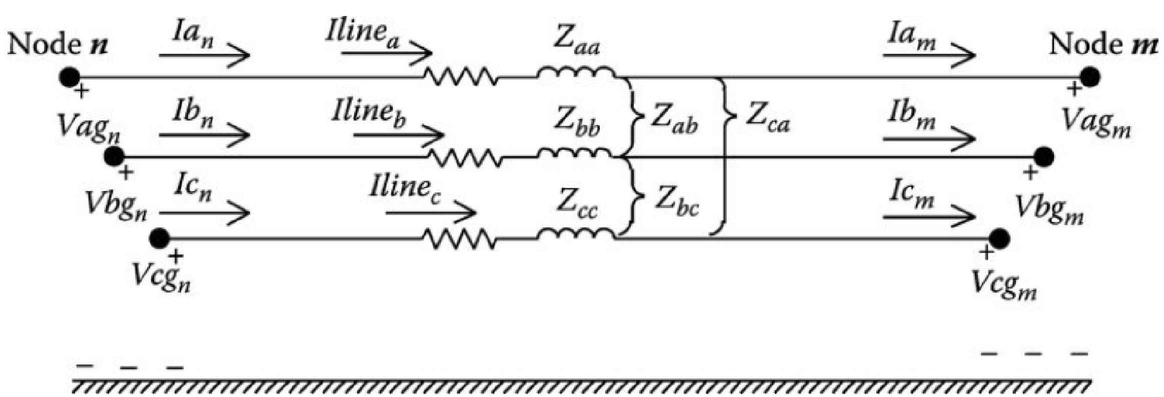
Distribution System Line Models – Overview

Highlights of This Section: there are three line segment models

Exact line segment model

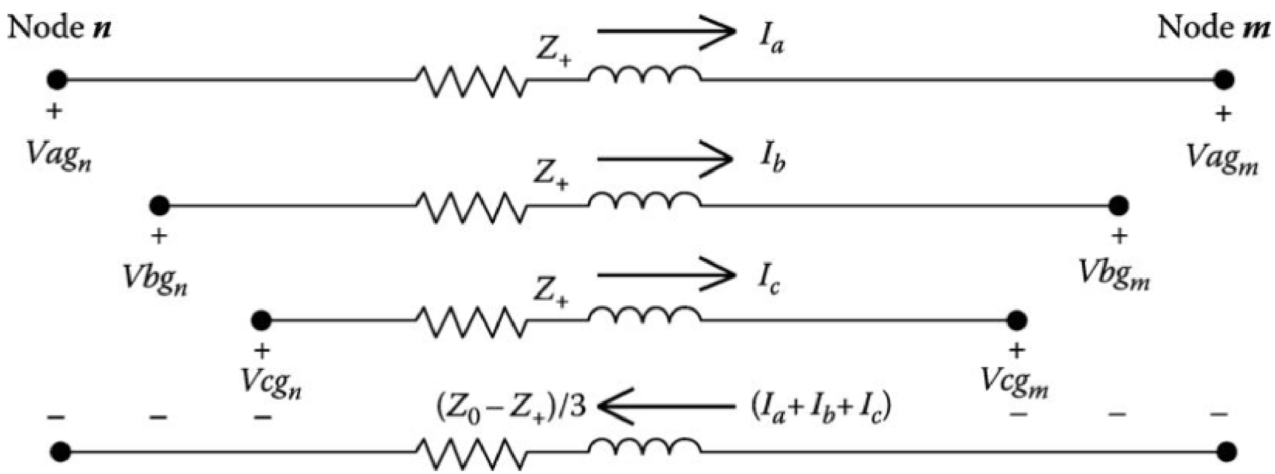


Modified line segment model (neglecting shunt admittance of exact model)



Distribution System Line Models – Overview

Approximate line segment model (in sequence domain)



- In this section, we will study how to derive the above three models (from KVL and KCL), i.e., how to derive forward-backward sweep models.

- Here we will assume [Zabc] and [Yabc] are known. How to compute the phase impedance and phase admittance matrices using the actual phasing of the line and the correct spacing between conductors are discussed in modeling series impedance and shunt admittance.

Exact Line Segment Model

The exact model of a three-phase, two-phase, or single-phase overhead or underground line is shown in Fig.1.

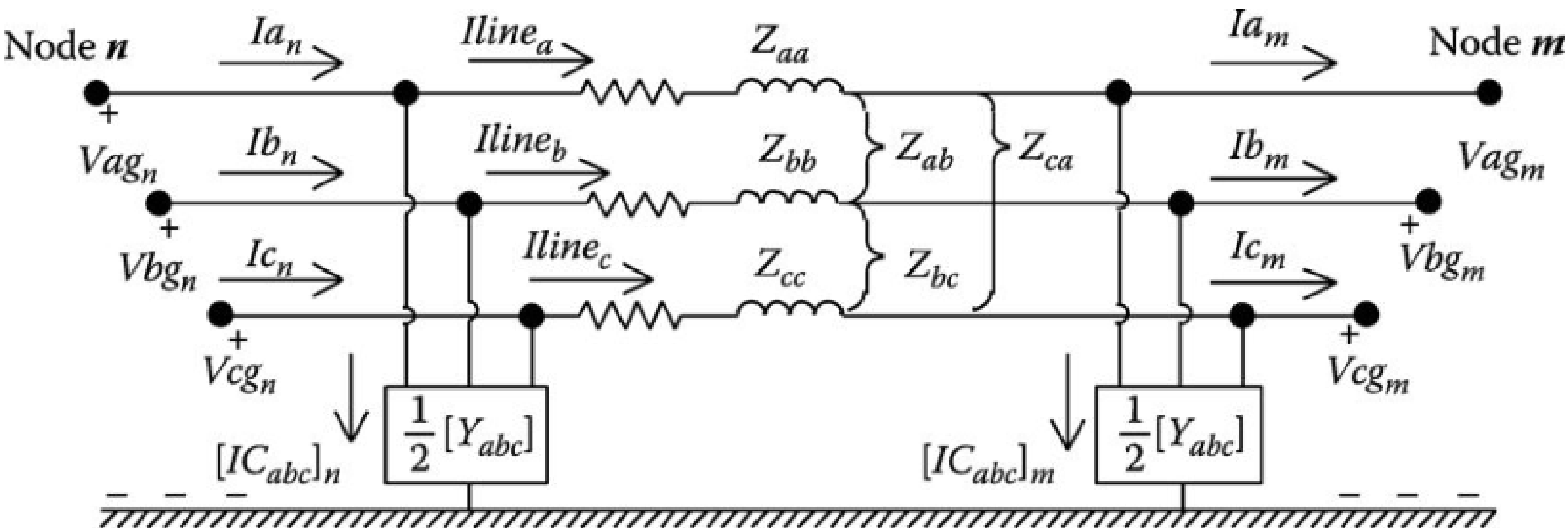


Fig.1 Three-phase line segment model

Exact Line Segment Model

- When a line segment is two phase (V phase) or single phase, some of the impedance and values will be zero. Note that in all cases the phase impedance and phase admittance matrices were 3 × 3. Rows and columns of zeros for the missing phases represent two-phase and single-phase lines. Therefore, one set of equations can be developed to model all overhead and underground line segments.
- The values of the impedances and admittances in Fig.1 represent the total impedances and admittances for the line segment. That is, the phase impedance/admittance matrix has been multiplied by the length of the line segment.

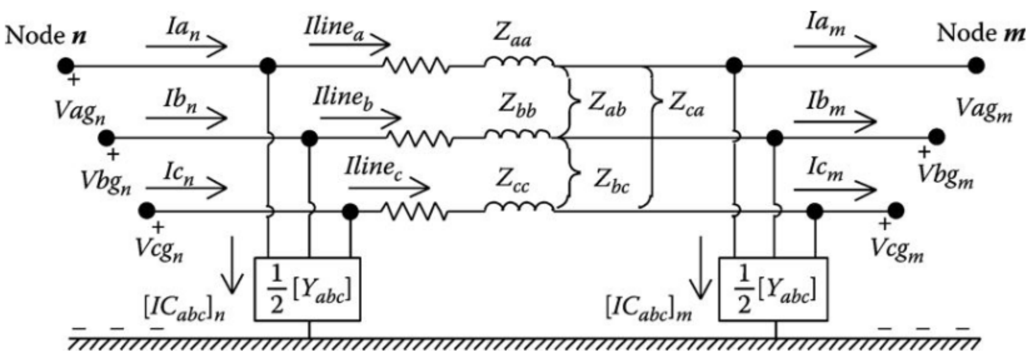


Fig.1 Three-phase line segment model

Exact Line Segment Model

For the line segment of Fig.1, the equations relating the input (node ***n***) voltages and currents to the output (node ***m***) voltages and currents are developed as follows.

Kirchhoff's current law applied at node ***m*** is represented by

$$\begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_m + \frac{1}{2} \cdot \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix} \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m \quad (1)$$

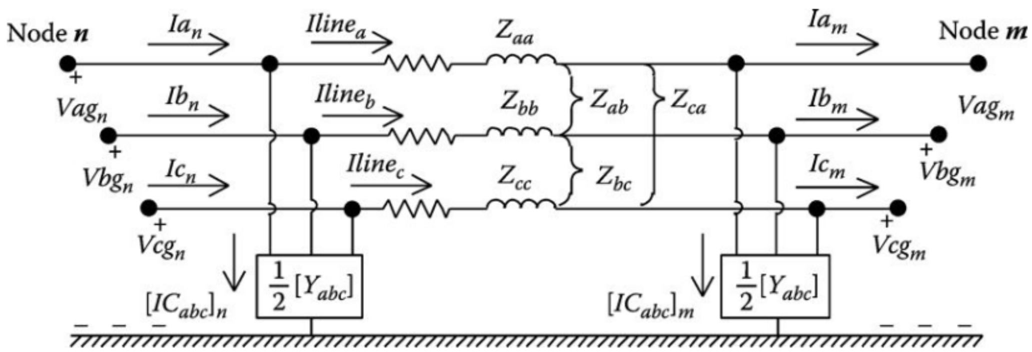


Fig.1 Three-phase line segment model

Exact Line Segment Model

$$\begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_m + \frac{1}{2} \cdot \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m \qquad (1)$$

In condensed form Equation (1) becomes

$$[Iline_{abc}]_n = [I_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_m \qquad (2)$$

Kirchhoff's voltage law applied to the model gives

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m + \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} \qquad (3)$$

In condensed form Equation (3) becomes

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}] \cdot [Iline_{abc}] \qquad (4)$$

Exact Line Segment Model

$$[Iline_{abc}] \; = [I_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_m \qquad (2)$$

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}] \cdot [Iline_{abc}] \qquad (4)$$

Substituting Equation (2) into Equation (4),

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}] \cdot \left\{ [I_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_m \right\} \qquad (5)$$

Collecting terms,

$$[VLG_{abc}]_n = \left\{ [u] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}] \right\} \cdot [VLG_{abc}]_m + [Z_{abc}] \cdot [I_{abc}]_m \qquad (6)$$

where

$$[u] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (7)$$

Equation (6) is of the general form

$$[VLG_{abc}]_n = [a] \cdot [VLG_{abc}]_m + [b] \cdot [I_{abc}]_m \qquad (8)$$

Exact Line Segment Model

$$[VLG_{abc}]_n = [a] \cdot [VLG_{abc}]_m + [b] \cdot [I_{abc}]_m \qquad (8)$$

where

$$[a] = [u] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}] \qquad (9)$$

$$[b] = [Z_{abc}] \qquad (10)$$

The input current to the line segment at node ***n*** is

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_n = \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n \qquad (11)$$

In condensed form, Equation (11) becomes

$$[I_{abc}]_n = [Iline_{abc}] + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_n \qquad (12)$$

Exact Line Segment Model

$$[Iline_{abc}] = [I_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_m \quad (2)$$

$$[I_{abc}]_n = [Iline_{abc}] + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_n \quad (12)$$

Substitute Equation (2) into Equation (12):

$$[I_{abc}]_n = [I_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_n \quad (13)$$

$$[VLG_{abc}]_n = \left\{[u] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}]\right\} \cdot [VLG_{abc}]_m + [Z_{abc}] \cdot [I_{abc}]_m \quad (6)$$

Substitute Equation (6) into Equation (13):

$$[I_{abc}]_n = [I_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \cdot [VLG_{abc}]_m + \frac{1}{2} \cdot [Y_{abc}] \left\{\left\{[u] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}]\right\} \cdot [VLG_{abc}]_m + [Z_{abc}] \cdot [I_{abc}]_m\right\} \quad (14)$$

Collecting terms in Equation (14),

$$[I_{abc}]_n = \left\{[Y_{abc}] + \frac{1}{4} \cdot [Y_{abc}] \cdot [Z_{abc}] \cdot [Y_{abc}]\right\} \cdot [VLG_{abc}]_m + \left\{[u] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}]\right\} \cdot [I_{abc}]_m \quad (15)$$

Exact Line Segment Model

$$[I_{abc}]_n = \left\{[Y_{abc}] + \frac{1}{4}\cdot[Y_{abc}]\cdot[Z_{abc}]\cdot[Y_{abc}]\right\}\cdot[VLG_{abc}]_m + \left\{[u] + \frac{1}{2}\cdot[Z_{abc}]\cdot[Y_{abc}]\right\}\cdot[I_{abc}]_m \quad (15)$$

Equation (15) is of the form

$$[I_{abc}]_n = [c]\cdot[VLG_{abc}]_m + [d]\cdot[I_{abc}]_m \quad (16a)$$

where

Backward sweep equation

$$[c] = [Y_{abc}] + \frac{1}{4}\cdot[Y_{abc}]\cdot[Z_{abc}]\cdot[Y_{abc}] \quad (17)$$

$$[d] = [u] + \frac{1}{2}\cdot[Z_{abc}]\cdot[Y_{abc}] \quad (18)$$

$$[VLG_{abc}]_n = [a]\cdot[VLG_{abc}]_m + [b]\cdot[I_{abc}]_m \quad (8)$$

Equations (8) and (16) can be put into partitioned matrix form:

$$\begin{bmatrix}[VLG_{abc}]_n \\ [I_{abc}]_n\end{bmatrix} = \begin{bmatrix}[a] & [b] \\ [c] & [d]\end{bmatrix}\cdot\begin{bmatrix}[VLG_{abc}]_m \\ [I_{abc}]_m\end{bmatrix} \quad (19)$$

Exact Line Segment Model

$$\begin{bmatrix} [VLG_{abc}]_n \\ [I_{abc}]_n \end{bmatrix} = \begin{bmatrix} [a] & [b] \\ [c] & [d] \end{bmatrix} \cdot \begin{bmatrix} [VLG_{abc}]_m \\ [I_{abc}]_m \end{bmatrix} \tag{19}$$

Equation (19) is very similar to the equation used in transmission line analysis when the A, B, C, D parameters have been defined [1]. In the case here the a, b, c, d parameters are 3 × 3 matrices rather than single variables and will be referred to as the "generalized line matrices."

Equation (19) can be turned around to solve for the voltages and currents at node ***m*** in terms of the voltages and currents at node ***n***:

$$\begin{bmatrix} [VLG_{abc}]_m \\ [I_{abc}]_m \end{bmatrix} = \begin{bmatrix} [a] & [b] \\ [c] & [d] \end{bmatrix}^{-1} \cdot \begin{bmatrix} [VLG_{abc}]_n \\ [I_{abc}]_n \end{bmatrix} \tag{20}$$

The inverse of the a, b, c, d matrix is simple because the determinant is

$$[a] \cdot [d] - [b] \cdot [c] = [u] \tag{21}$$

[1] Glover, J.D. and Sarma, M., *Power System Analysis and Design*, 2nd edn., PWS Publishing Co., Boston, MA, 1995.

Exact Line Segment Model

$$\begin{bmatrix}[VLG_{abc}]_m \\ [I_{abc}]_m\end{bmatrix} = \begin{bmatrix}[a] & [b] \\ [c] & [d]\end{bmatrix}^{-1} \cdot \begin{bmatrix}[VLG_{abc}]_n \\ [I_{abc}]_n\end{bmatrix} \qquad (20)$$

$$[a] \cdot [d] - [b] \cdot [c] = [u] \qquad (21)$$

Using the relationship of Equation (21), Equation (20) becomes

$$\begin{bmatrix}[VLG_{abc}]_m \\ [I_{abc}]_m\end{bmatrix} = \begin{bmatrix}[d] & -[b] \\ -[c] & [a]\end{bmatrix} \cdot \begin{bmatrix}[VLG_{abc}]_n \\ [I_{abc}]_n\end{bmatrix} \qquad (22)$$

Since the matrix $[a]$ is equal to the matrix $[d]$, Equation (22) in expanded form becomes

$$[VLG_{abc}]_m = [a] \cdot [VLG_{abc}]_n - [b] \cdot [I_{abc}]_n \qquad (23)$$

$$[I_{abc}]_m = -[c] \cdot [VLG_{abc}]_n + [d] \cdot [I_{abc}]_n \qquad (24)$$

Exact Line Segment Model

$$[VLG_{abc}]_n = [a] \cdot [VLG_{abc}]_m + [b] \cdot [I_{abc}]_m \qquad (8)$$

Sometimes it is necessary to compute the voltages at node ***m*** as a function of the voltages at node ***n*** and the currents entering node ***m***. This is useful in the ladder iterative technique, i.e., the forward sweep equation.

Solving Equation (8) for the bus m voltages gives

$$[VLG_{abc}]_m = [a]^{-1} \cdot \{[VLG_{abc}]_n - [b] \cdot [I_{abc}]_n \ \} \qquad (25)$$

Equation (25) is of the form

$$[VLG_{abc}]_m = [A] \cdot [VLG_{abc}]_n - [B] \cdot [I_{abc}]_m \qquad (26)$$

where

$$[A] = [a]^{-1} \qquad (27)$$

$$[B] = [a]^{-1} \cdot [b] \qquad (28)$$

Exact Line Segment Model

The line-to-line voltages are computed by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_m = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m = [D] \cdot [VLG_{abc}]_m \tag{29}$$

where

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{30}$$

Because the mutual coupling between phases on the line segments is not equal, there will be different values of voltage drop on each of the three phases. As a result the voltages on a distribution feeder become unbalanced even when the loads are balanced. A common method of describing the degree of unbalance is to use the National Electrical Manufactures Association (NEMA) definition of voltage unbalance as given in Equation (31) [2].

$$V_{unbalance} = \frac{|Maximum\ Deviation\ from\ Average|}{|V_{average}|} \cdot 100\%$$

[2] ANSI/NEMA Standard Publication No. MG1-1978, National Electrical Manufactures Association, Washington, DC.